coils and inertia wheels or only coils is both feasible and desirable.

The electromagnetic actuation system requires less volume and weight and has greater reliability than other means of actuation, such as mass dispensing with cold gas or bipropellants. It also compares favorably to the gravity gradient and solar pressure techniques when the overall attitude control system is considered.

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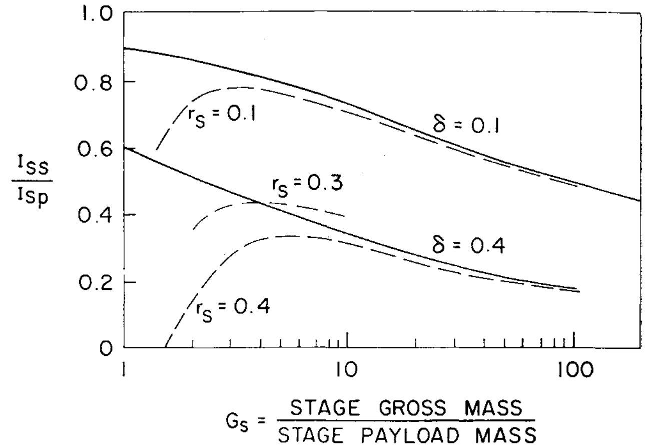

Fig. 1 Variation of I_{ss}/I_{sp} with different structural factors

Structural Factors and Optimization of Space Vehicles

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In the analysis of space vehicles, structural factors—originally defined as means of identifying structural mass—have been more recently treated as functions. This gives them a quite different connotation.

In 1947, Malina and Summerfield[1] defined the structural factor as the ratio of stage mass ejected, m_s, to stage mass with propellant, $\delta = m_s/(m_s + m_p)$, and, so defined, it pertains only to one particular stage, and involves neither the payload of that stage nor the payload or mass of any subsequent stages. Another factor alternately used is $r_s = m_s/(m_s + m_p + m_l)$, m_l being the stage payload; this factor includes all subsequent stages and their payloads.

Both factors are used by some authors,[2, 3] the choice depending upon the application, and both are convenient *realistic* terms used to identify particular structures. In analyses they are used as parameters, modifications in design being handled by usual methods of variable parameters or parametric curves.

However, in recent analyses,[4, 5] for the optimum number of stages, n_o, the optimum is assumed to be at $\partial\delta/\partial n$(or $\partial r_s/\partial n$) $= 0$, and δ (or r_s) is used as a function determined by n. With this usage δ (or r_s) is no longer an identifying parameter but a function describing a particular manner of variation of structural hardware mass with variation of n—e.g., $m_s = (\text{constant})(m_s + m_p)$.

To use the factor in this sense implies a foreknowledge of the functional influence of all the complex factors which determine m_s. Since it would be a monumental task to determine these precisely, it is pertinent to learn how sensitive important vehicle characteristics are to certain structural factors used as functions.

The system specific impulse, I_{ss}—introduced in a previous paper[3] as a measure for comparing propulsive systems that have wide differences in mass, and expressed in terms of δ by Eq. (11) of Ref. 3—for a single-stage vehicle becomes, with the aid of Eqs. (4) and (6) of the same reference,

$$I_{ss} = I_{sp}\{1 - [\log (1 + r_sG_s)]/\log G_s\} \quad (1)$$

where $G_s = (m_s + m_p + m_{ls})/m_{ls}$. At $\partial I_{ss}/\partial G = 0$, where $G_s = G_{so}$, $(I_{ss})_{\max} = 1/(1 + r_sG_{so})$. This depends only upon r_s and G_{so}, and is the same[3] as that for a multistage missile with proportionate staging.[1] Fig. 1 shows, after adjusting for $r_s = \delta(1 - G_s^{-1})$, that the essential difference introduced between the two curves obtained by treating r_s or δ as constant, enters only for $G < 3$.

For proportionate multistaging, I_{ss} can be given either by Eq. (24) of Ref. 3 in terms of δ and n, or by $I_{ss} = -I_{sp}(c/n)/\log \{e^{-c/n} - r_s\}$ in terms of r_s and n. These equations, and a similar equation in terms of two structural factors defined by Cooper[6] for $m_s = \epsilon m_o + fm_p$, where the first term is the "engine" and the second the "tankage" mass, are all of the form

$$I_{ss}/I_{sp} = -(c/n)/[\log (Ae^{-c/n} - B)] \quad (2)$$

where the constants A and B are determined by the structural functions δ, r_s, or ϵ and f.

Similarly, the gross mass ratio m_o/m_l is given, respectively, by

$$G = (1 - \delta)^n(e^{-c/n} - \delta)^{-n} = (e^{-c/n} - r_s)^{-n} = [(1 + f)e^{-c/n} - (\epsilon + f)]^{-n}$$

where $c = v_r/GI_{sp}$, or in general by

$$G = R(e^{-c/n} - S)^{-n} \quad (3)$$

where the quantities R and S are determined by δ,r_s, or ϵ and f.

Eq. (2) has a maximum, which for the form with δ is at $n \to \infty$ and for the other structural functions is at a finite n. Similarly, Eq. (3) has a minimum at n_m ranging from a small finite integer (5 in Fig. 2) to infinity, depending upon the choice of structural function. Because of this sensitivity to choice of structural function, the optimum n_o—i.e., the number of stages to give minimum G—is seen to be dependent more upon the selection of this function than upon the optimizing procedure, and to this extent is indeterminant.

Instead, if we observe that G decreases sharply from an infinite value where n is minimum, and levels off to a broad minimum (Fig. 2), regardless of the structural function used, we can define a preferred[3] minimum number of stages determined at the keen of the curve rather than the optimum at minimum G. In Fig. 2 this gives $n = 3$ or 4 for the preferred n rather than a range from $n = 5$ to infinity for the minimum G. In any practical application the small increase in G obtained by reducing n from 5 to 3 or 4 is more than offset by the reduction in complexity resulting from fewer stages. The preferred number, n_p, being independent of the choice of structural function, is then more closely related to practical design circumstances and is more definitive.

We can locate n_p at the knee of Eq. (3) from the curve graphically or from the integral portion of $n_p = 1 + (K - 1/\log S)c$. The term $-1/\log S$ is derived from the minimum

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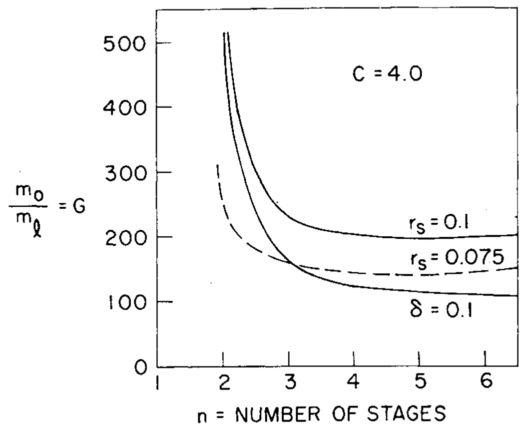

Fig. 2 Variation of mass ratio with different structural factors

number of stages given by $e^{-c/n} - S = 0$. The constant Kc is added as a first linear correction for the horizontal stretching of the knee introduced by an increasing c, with K selected from observation to be 0.35 for typical cases.

It is concluded on the basis of the foregoing that the use of structural factors as functions in optimization analyses introduces the necessity of using either (1) a more definitive structural function when optimizing for minimum mass or (2) a more definitive optimization procedure such as the one suggested above based on the logarithmic nature of the equations. Because of the difficulties of the definitive specification of a structural function, the latter alternative appears preferable.

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On Transient Magnetohydrodynamic Flow in Channels

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In a recent paper Yen and Chang[1] have, using the Laplace transform, studied the magnetohydrodynamic flow in a channel of perfectly conducting walls with a transverse magnetic field and a sudden change of the axial pressure gradient. It is the purpose of this note to investigate the flow in a channel of insulating walls by another method for any change in the time-dependent axial pressure gradient, and also to point out a feature not adequately discussed by Yen and Chang. The velocity field and the induced electric (and magnetic) field are of the oscillatory type with decaying amplitudes, particularly when the magnetic Prandtl number is nearly unity.

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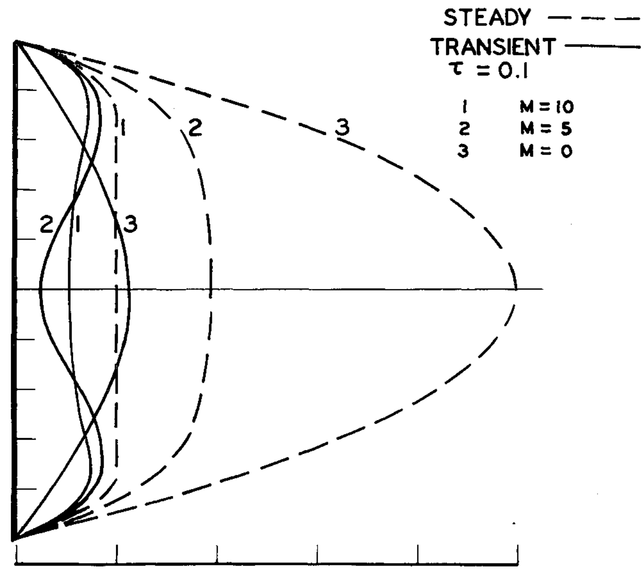

Fig. 1 Velocity profiles with magnetic Prandtl number = 1, $(\rho\nu/P_1h^2)u$ vs $\xi(= y/h)$

Taking x-axis along the channel, the governing equations of one-dimensional unsteady flow are

$$\frac{\partial u}{\partial t} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{P(t)}{\rho} + \frac{\sigma}{\rho}B_0E_z - \frac{\sigma B_0^2}{\rho}u$$

$$\frac{\partial E_z}{\partial t} + B_0\frac{\partial u}{\partial t} = \lambda\frac{\partial^2 E_z}{\partial y^2} \qquad (1)$$

$$\frac{\partial B_x}{\partial t} - \lambda\frac{\partial^2 B_x}{\partial y^2} = B_0\frac{\partial u}{\partial y}$$

where P is the time-dependent axial pressure gradient. The initial and boundary conditions are

$$t = 0:\quad u = u_s(y),\ P = P_s = \text{const},\ E_z = E_s = \text{const}$$
$$t > 0;\ y = \pm h:\quad u = 0,\ P = P_1(t) + P_s,\ E_z = E_1(t) + E_s \qquad (2)$$

where the subscript s denotes the steady state. E_s and E_1 are unknown a priori, they must be found from the integration of current density,

$$t \geqslant 0:\quad \int_{-h}^{h} j_z\,dy = \int_{-h}^{h}(E_z + uB_0)\,dy = 0 \qquad (3)$$

However, owing to the property of symmetry the instantaneous electric field at both walls $y = \pm h$ must be the same.

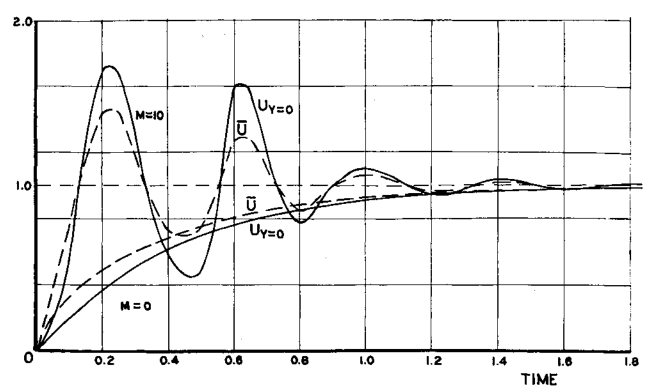

Fig. 2 Time history of $U = u/u_s$ (magnetic Prandtl number = 1)